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A SURVEY OF SOME MODELS FOR DETERMINING MUNITIONS STOCKPILE REQ--ETC(U)

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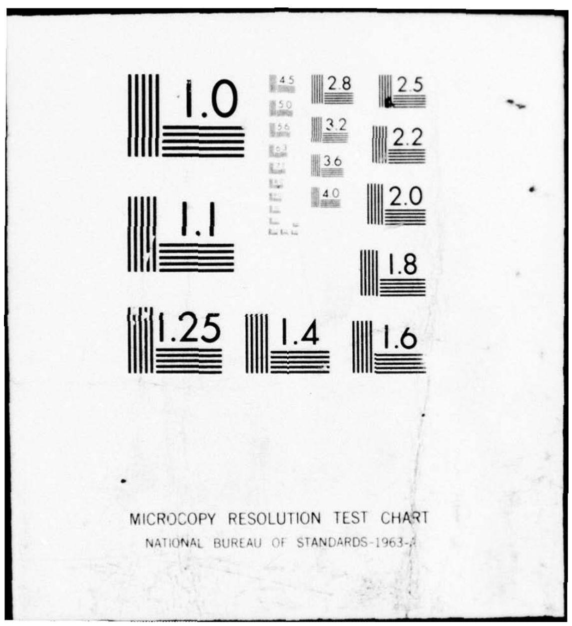

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

A SURVEY OF SOME MODELS FOR

DETERMINING MUNITIONS STOCKPILE

REQUIREMENTS FOR AIR TO GROUND WEAPONS

by

James K. Hartman

January 1978

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Prepared for
Naval Weapons Center
China Lake, California 93555

NAVAL POSTGRADUATE SCHOOL
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Five models for determining munitions requirements for air-to-ground weapons are compared. Common features and differences are discussed, and suggestions for extensions are presented.

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A SURVEY OF SOME MODELS FOR DETERMINING MUNITIONS STOCKPILE REQUIREMENTS FOR AIR TO GROUND WEAPONS

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TABLE OF CONTENTS

ABSTRACT

Five models for determining munitions requirements for air-to-ground weapons are compared. Common features and differences are discussed, and suggestions for extensions are presented.

I. INTRODUCTION

A. The Munitions Planning Problem

The problem of determining the size of munitions stockpiles requires decision makers to assess the difficult tradeoff between the high costs of acquiring munitions and keeping them in inventory versus the risk of inadequate capability if the inventory is too small. This tradeoff is made more difficult by several factors.

1. Large amounts of money are involved, particularly because of the increasing unit costs of new sophisticated weapons.
2. The increasing complexity of these weapons implies longer time delays before production can be resumed in the event of a major contingency. Hence inventories may have to be larger.
3. The high rate of technological progress implies that today's munitions inventory may be obsolete tomorrow. The existence of a large and expensive inventory of today's weapons may make tomorrow's decision to buy better weapons and munitions more difficult leading to a technologically inferior force.

In a discussion paper by Sovereign [1] these issues are explored further and the munitions stockpile problem is related to the longer range force structure problem of determining what weapons systems and platforms to develop.

This paper concentrates on the more tractable short range problem of determining munitions requirements for given engagement scenarios. We assume that the delivery platforms,

the types of munitions they can deliver, and the engagement scenario are fixed. The decisions to be made are:

1. the amount of each munitions type required.
2. the allocation of each munitions type to delivery platforms of each type, and hence to the targets in the scenario.

Several models exist for computing munitions requirements under the above assumptions. The models vary greatly in degree of detail, flexibility, and complexity. The primary purposes of this paper are to present a survey of existing models for determining air-to-ground munitions requirements and to highlight major issues in the development of such models.

B. The Models to be Considered

Several existing models developed for the Navy and the Air Force address the air-to-ground munitions requirement problem. In this paper we will survey the models presented in the following references:

1. RAND LINEAR PROGRAM (1971). J. Y. Lu and R. B. S. Brooks, "WRM Requirements Computation for the Air Force Nonnuclear Air-to-Ground Munitions, Volume 1: A Model," RAND Corp., R-800/1-PR, October 1971, (reference 2).
2. RAND NONLINEAR PROGRAM (1974). R. J. Clasen, G. W. Graves, and J. Y. Lu, "Sortie Allocation by a Nonlinear Programming Model for Determining a Munitions Mix," RAND Corp., R-1411-DDPAE, March 1974, (reference 3).

3. TAC RESOURCER (1976). R. P. Harvey, R. D. McKnight, and G. B. Dantzig, "Development and Implementation of TAC RESOURCER: A Large Scale Ordnance Planning and Resources Allocation Computer System," Control Analysis Corporation, May 1976, (reference 4).

4. NAVMOR (1974). Kent I. Johnson, "Documentation of NAVMOR FY'74 Computer Program," Naval Weapons Center Tech Note 12-74-1, Naval Weapons Center, China Lake, June 1974, (reference 6).

5. NAVMOR PLUS (1976). S. S. Bloom, "NAVMOR PLUS Users Manual," NAVCOSSACT Document 53E234, UM-01, Naval Command Systems Support Activity, (reference 8).

The first three of these models were developed for the Air Force, while the last two are Navy models. All except the NAVMOR model are optimization models using either linear or nonlinear programming techniques to find a "best" solution to the munitions requirement problem.

Section II of this report presents a summary of each of the above models. In Section III we compare the models by considering each of the major components of a munitions requirements model, and present some additional ideas which might be incorporated into future models. Section IV summarizes the report.

II. CHARACTERISTICS OF SEVERAL MUNITIONS REQUIREMENTS MODELS

All of the models discussed in this report have certain features in common. Each starts with a combat scenario expressed as a list of enemy targets or target types to be defeated. Limited resources are available in the form of aircraft sorties for various types of aircraft and various delivery conditions (e.g. day/night). The availability of each type of sortie is considered fixed in the short run scenario. The decisions to be made include which munitions to use against each target and which aircraft sortie type to use to deliver the selected munition. Munition availiability is assumed unlimited. The models then aggregate the total munitions used to compute the requirements for this scenario.

Each munition and sortie allocation has several measures of effectiveness associated with it. The cost of munitions and (perhaps) the cost of attrition to aircraft are summarized for each allocation considered. The expected number or value of targets destroyed is also computed to provide the combat effectiveness which is purchased at the indicated cost.

Most of the models are optimizing models which select munitions to either minimize the cost to achieve specified levels of destruction or maximize target value destroyed subject to a budget constraint.

Within this common model structure there is room for substantial differences in level of detail, computation of effectiveness, factors included or excluded from the analysis,

and ease of solution of the resulting model. We will now summarize each of the models in some detail to show some of the possible variations.

A. RAND LINEAR PROGRAM (1971).

This model [2], developed by the RAND Corporation for the U. S. Air Force, uses linear programming to select a minimum cost munitions buy. The munitions selection must satisfy constraints on targeting requirements and on limited sortie availability in each of several contingencies. Sortie availability is further reduced if the model selects munition-sortie-target combinations which result in aircraft attrition.

1. Decision Variables. There are two classes of decision variables in this model:

   a. $x_{ijk\ell}$ = the number of sorties flown by aircraft of type $i = 1,\dots,m$ against targets of type $j = 1,\dots,n$ using munitions of type $k = 1,\dots,p$ in contingency $\ell = 1,\dots,q$.

   b. y_k = the total amount of munitions of type k required.

2. Objective Function. The model has two possible objective functions either of which may be used.

   a. Minimize munitions cost,

$$\min \sum_k f_k y_k$$

   where f_k is the constant unit cost of munitions of type k.

b. Minimize munitions cost plus sortie cost (including attrition cost)

$$\min \sum_{k} f_k y_k + \sum_{ijk\ell} g_{ijk\ell}\, x_{ijk\ell}$$

where $g_{ijk\ell}$ is a constant cost per sortie including attrition but <u>not</u> including cost of the munitions load.

3. <u>Target Damage Constraints</u>. The solution is required to meet specified damage probabilities for each target type in each contingency by a linear constraint:

$$\sum_{ik} b_{ijk\ell}\, x_{ijk\ell} \leq c_{j\ell} \qquad \forall\ j,\ \ell$$

where $b_{ijk\ell}$ and $c_{j\ell}$ are constants computed from the required damage levels. Note that the constraint is $\leq$ because the coefficient $b_{ijk\ell}$ is related to the probability of surviving the attack.

4. <u>Sortie Availability Constraints</u>. The limitations on sortie availability and the effects of aircraft attrition are reflected in aircraft availability constraints.

$$\sum_{jk} d_{ijk\ell}\, x_{ijk\ell} \leq e_{i\ell} \qquad \forall\ i,\ \ell$$

where $d_{ijk\ell}$ is an attrition factor (≥ 1) which indicates that in order to achieve $x_{ijk\ell}$ sorties actually arriving at the target, we must schedule more than $x_{ijk\ell}$ sorties.

The right hand side value $e_{i\ell}$ is the number of sorties available by aircraft i in contingency ℓ.

5. Munitions constraints. The RAND linear programming model may consider several contingencies simultaneously. For each contingency the $x_{ijk\ell}$ are computed as above. The actual munitions requirement y_k is constrained to be great enough to handle any single contingency (but not necessarily two or more contingencies) by the constraints:

$$y_k - \sum_{ij} a_{ijk\ell}\, x_{ijk\ell} \geq 0 \qquad \forall\ k, \ell$$

where $a_{ijk\ell}$ is the normal munitions load for a sortie of type $ijk\ell$.

6. Model Summary.

$$\text{minimize} \quad \sum_k f_k y_k$$

$$\begin{aligned} \text{subject to} \quad & y_k - \sum_{ij} a_{ijk\ell}\, x_{ijk\ell} \geq 0 && \forall\ k, \ell \\ & \sum_{ik} b_{ijk\ell}\, x_{ijk\ell} \leq c_{j\ell} && \forall\ j, \ell \\ & \sum d_{ijk\ell}\, x_{ijk\ell} \leq e_{i\ell} && \forall\ i, \ell \\ & x_{ijk\ell} \geq 0 && \forall\ ijk\ell \end{aligned}$$

Excluding any slack variables and all the nonnegativity constraints, the model has $mnpq + p$ variables and $pq + nq + mq$ constraints.

7. Solution Method. The RAND study proposes the Dantzig-Wolfe decomposition principle for solution of the problem.

8. Miscellaneous Notes.

   a. Variations in weather and/or delivery tactics can be handled through the sortie availability constraints by letting the aircraft subscript i take on multiple values for each aircraft corresponding to that aircraft under varying delivery conditions. Of course the sortie availabilities $e_{i\ell}$ must then also be broken down by delivery condition.

   b. The input data for this form of model must include the desired probability of defeating each class of targets.

   c. It is noted in the source report [2] that the treatment of sortie attrition cost in the model is not logically consistent with the multicontingency formulation.

   d. The treatment of target damage in a linear constraint requires some assumptions about how sorties are allocated to targets within each i, j class. For details see reference 2, page 19.

B. RAND NONLINEAR PROGRAM (1974).

This model [3] formulated by the RAND Corporation, uses a nonlinear programming approach to maximize expected military worth of targets killed. The optimal allocation of sorties must satisfy constraints on sortie availability and bounds on the expected number of targets killed.

1. Decision Variables. The decision variables for this model are

s_{ij} = the number of sorties flown by aircraft of type $i = 1,\ldots,m$ against targets of type $j = 1,\ldots,n$.

(The munitions load for each sortie target combination is fixed having been previously selected to minimize cost per expected kill.)

2. Objective Function. The model chooses the sortie allocation to maximize expected military worth of the targets killed. The number of kills K_j for targets of type j is given as a nonlinear diminishing returns function of S_{ij} as

$$K_j = \frac{T_j}{C_j}\left\{1 - \exp\left[-\frac{C_j}{T_j}\left(\alpha_j + \sum_i P_{ij}S_{ij}\right)\right]\right\}$$

where

$$\alpha_j = \left(-\frac{T_j}{C_j}\log 1 - \frac{C_j}{T_j}D_j\right),$$

and where

T_j = number of type j targets,

D_j = number of type j targets already killed,

C_j = a parameter for each target type designating the extent to which dead targets can be distinguished from live ones, and hence controlling the extent to which diminishing returns applies,

and P_{ij} = number of type j targets killed by a type i sortie.

Then the objective function is

$$\text{maximize} \quad \sum_j V_j (K_j - D_j)$$

where V_j = the value or military worth of each type j target

and K_j = the nonlinear kill function defined above.

3. Target Damage Constraints. Given the function K_j, bounds on target damage are defined as

$$\ell_j \leq K_j \leq T_j \qquad \forall j .$$

The lower bounds specify minimum required damage to targets of type j, while the upper bound T_j makes sure that no more than T_j targets are killed (without this bound the objective function might try to accumulate value for targets that do not exist). As indicated in [3] the constraints of this form can be transformed to be linear in S_{ij}.

4. Sortie Constraints. Upper and lower bounds on sortie utilization take the form

$$\sum_{j\in J} S_{ij} \underset{(\geq)}{\leq} f_J \sum_{j=1}^{n} S_{ij} \quad \forall\ i \text{ (perhaps several different sets } J \text{ for each } i)$$

to place limits on the sorties flown by aircraft of type i against the subset of targets included in a set J as a fraction f_J of total sorties flown by type i aircraft. The total sorties flown by each aircraft type are also limited by the constraint

$$\sum_j S_{ij} = S_i \qquad \forall\ i$$

where S_i is the total available.

5. Model Summary.

$$\begin{array}{lll} \text{maximize} & \sum_j V_j (K_j - D_j) & \\ \text{subject to} & \ell_j \leq K_j \leq T_j & \forall\ j \\ & f_J \sum_j S_{ij} \leq \sum_{j\in J} S_{ij} \leq g_J \sum_j S_{ij} & \forall\ i, J \\ & \sum_j S_{ij} = S_i & \forall\ i \\ & S_{ij} \geq 0 & \forall\ i,j \end{array}$$

where K_j is the nonlinear kill function as defined earlier. If the damage constraints are linearized the resulting model has a nonlinear objective function with linear constraints. The model has mn variables and approximately $2n + 2ms + m$ constraints (depending on the average number S of target subsets J for each i).

6. Solution Method. A nonlinear programming algorithm by Graves [3] is used to solve the model.

7. Miscellaneous Notes.

a. The RAND NONLINEAR model does not optimize the munitions selection for each sortie-target combination in the NLP model. Instead, the munitions are preselected on a least-cost-per-expected-kill basis for each aircraft-target combination. The effect of this preselection is to make it impossible for sortie limitations to have an effect on munitions selection. For further discussion of this matter see Section III.

b. Cost is considered only in the munition preselection process described above, and does not appear in the sortie allocation model at all. The costs considered in the munitions preselection may include attrition cost.

c. The total munitions requirement is computed subsequent to optimization by combining the optimal S_{ij} with the preselected munitions load for each i, j combination.

d. Sortie attrition does not appear explicitly in the model. The source report [3] suggests that the diminishing returns function K_j includes provision for attrition, but this can only be at the most primitive level since K_j includes only one tuning parameter, C_j, for each target class. This parameter is independent

of the aircraft i (and hence the chosen munition), so that attrition, if it is modeled at all, is a function only of the target, not of the attacking aircraft-munition combination.

e. The fractional sortie constraints would seem to be somewhat ad hoc.

f. The model requires input values V_j for the military worth of each target class. Thus the user must explicitly determine the value of (say) a fuel storage location relative to that of a population center.

C. TAC RESOURCER (1976). TAC RESOURCER [4,5] is the most recent Air Force model for determining air-to-ground munitions requirements. It is considerably more complex than either of the two RAND models. The model uses a large scale nonlinear optimization to maximize the total expected military worth of targets killed. The optimal sortie allocation must satisfy constraints on sortie attrition, sortie and munitions cost, targets killed, and other restrictions on various combinations of aircraft and targets.

1. Decision Variables. The decision variables for this model are

$x_{ijkd\ell t}$ = the number of sorties flown by aircraft of type i using ordnance of type j against targets of type k, the sortie being flown with delivery condition d, in weather state ℓ, and in time period t.

From this definition it is clear that this model considers more alternative cases than either of the RAND models.

2. Objective Function. The model chooses the sortie allocation to maximize the expected military worth of the targets killed

$$\max \sum_{t} \sum_{k} \frac{V_{kt} T_{kt}}{C_{kt}} \left\{ 1 - \exp\left[- \frac{C_{kt}}{T_{kt}} \sum_{ijd\ell} E_{ijkd}\, x_{ijkd\ell t} \right] \right\}$$

where

V_{kt} = target value

T_{kt} = total number of targets

C_{kt} = a factor that accounts for targets killed previously ($0 < C_{kt} \leq 1$)

E_{ijkd} = expected number of target kills for a sortie with indices $ijkd\ell t$ assuming it is not competing with other sorties and that targets are available.

The expression is similar to that of the RAND nonlinear program.

The model has the capability of handling up to four objectives in a hierarchical manner. The other three objectives possible are total number of sorties flown, total aircraft attrition and total aircraft and weapon cost.

3. Sortie Availability Contraints. The limited availability of aircraft is reflected in the sortie constraints:

$$\sum_{jkd\ell} x_{ijkd\ell t} \leq \hat{S}_{it} \qquad \forall\ i,t$$

4. Potential Target Kills Constraints. Upper and lower bounds on target damage are given by

$$Q'_{kt} \leq \sum_{\tau=1}^{t} \sum_{ijd\ell} E_{ijkd} \, x_{ijkd\ell\tau} \leq H'_{jt} \qquad \forall \; k, t.$$

5. Limit on Acceptable Attrition. Aircraft attrition is limited by the constraint:

$$\sum_{\tau=1}^{t} \sum_{jkd\ell} \hat{A}_{ijkd\tau} \, x_{ijkd\ell\tau} \leq A'_{it} \qquad \forall \; i, t$$

where $\hat{A}_{ijkdt}$ = expected attrition rate for sorties of type $ijkd\ell t$.

6. Aircraft/Weather Sortie Constraints. Limits on sorties flown in given weather states are given by

$$\sum_{jkd} x_{ijkd\ell t} \leq (\text{or} = , \text{ or} \geq) \; R_{i\ell t} \qquad \forall \; i, \ell, t$$

7. Target Availability/Weather Constraints. Limits on potential target kills in various weather states are given by

$$\sum_{jkd} E_{ijkd} \, x_{ijkd\ell t} \; (\leq, =, \text{ or } \geq) \; T_{k\ell t} \qquad \forall \; k, \ell, t$$

8. Ordnance Constraints. Limits on ordnance use can be imposed by

$$O_{1jt} \leq \sum_{\tau=1}^{t} \sum_{ikd\ell} w_{ijc} \, x_{ijkd\ell\tau} \leq O_{2jt} \qquad \forall \; j, t$$

9. Operational Judgment Constraints. Aircraft/target combinations can be restricted by constraints

$$\sum_{jd\ell} x_{ijkd\ell t} \; (\leq, =, \text{ or } \geq) \; G_{ikt}.$$

10. Budgetary Constraints. Nonlinear cost constraints for each time period can be imposed for each aircraft type and for each ordnance type, for example, aircraft cost is constrained by

$$\bar{C}_{lit}\Big(\sum_{jkd\ell} x_{ijkd\ell t}\Big) \leq B_{lit}$$

where $\bar{C}_{lit}$ is a convex function. Attrition cost is not included.

In addition one overall budget constraint which combines aircraft and ordnance cost, may be imposed for each time period, and cost constraints can be imposed for combinations of ordnance types (e.g. all missiles). All cost constraints are similar to the above example in that they assume convex cost functions.

11. Solution Methods. The model resulting from the above relationships is large, nonlinear, and complex. The solution is computed suboptimally one time period at a time, with the results from earlier time periods helping to define the constraint bounds for later periods. The nonlinear problem for each time period is approximated using piecewise linear functions yielding a linear program

which typically may have 250 constraint rows and 100,000 variables. Each L.P. is solved using a large scale linear programming package, the CAMPS program [4, pages 2-31].

12. Miscellaneous Notes.

a. This model is large and complex, and incorporates a number of optional objective and constraint features. The formulation requires specification of a large number of constraint bound values.

b. The model requires input values V_j for the military worth of each target type.

c. The requirement for convex cost functions in the budgetary constraints seems to stem from computational ease (they yield a convex NLP which is feasible to optimize using the selected algorithm), rather than from model formulation and principles. If the costs are not linear, we would expect a concave function to better reflect the real cost environment where economies of scale and learning curves imply decreasing average cost as procurement quantities increase.

D. NAVMOR (1974). The NAVMOR model (Navy/Marine Corps Ordnance Requirements) [6] is unique among those surveyed in this report because it does not attempt to allocate aircraft sorties to targets in a way that optimizes any objective function. Instead, the sortie allocation is determined from the scenarios by the subjective judgment of Naval officers. Thus the model

produces a typical outcome for the conflict scenario rather than the best that could be achieved if all decisions were optimized.

1. Variables and Subscripts. Variables and constants in the NAVMOR model have subscripts similar to those of the other models:

   i = aircraft type

   j = weapon type

   k = target type

   d = delivery tactic

   ℓ = ceiling level

   m = mission type

   r = weapon load (N) and number of passes (n).

2. Preliminary Calculations. The following probabilities are input or computed for each NAVMOR run.

   a. $P_T(i,j,k,d,\ell,m,r)$ = probability of penetration to the target area for a sortie of type (i,j,k,d,ℓ,m,r).

   b. $P_{sy}(i,j,k,d,\ell,m,r)$ = given penetration, probability of survival up to and including the y^{th} pass, $y = 1,2,\ldots, n(r)$.

   c. $P_R(i,j,k,d,\ell,m,r)$ = probability of successful return given that ordnance dropped over the target and survived all passes.

   d. $P_K(i,j,k,d,\ell,m,r)$ = single pass probability of target kill, computed using formulas from the Joint Munitions Effectiveness Manual.

e. N(j,r,d) = number of weapons of type j used in a sortie under conditions r, d.

3. <u>Measures of Effectiveness</u>. For each combination of subscripts (which we suppress for typographical convenience) NAVMOR computes

a. Weapon effectiveness:

$$E = P_T P_K \sum_{y=1}^{n} y(P_{sy} - P_{sy+1})$$

where y = number of passes (and hence the number of targets shot at) and $P_{s_{n+1}} = 0$.

b. Aircraft attrition:

$$A = 1 - P_T P_{sn} P_R$$

c. Sortie cost, including cost of weapons and aircraft attrition:

$$C = AC_k + C_m + NC_w$$

where

C_k = aircraft cost,

C_m = cost of a sortie (maintenance, fuel)

C_w = cost of each weapon for this sortie type

and N = number of weapons used in this sortie type.

Next, for each combination of subscripts, the following three measures of effectiveness are computed:

a. Cost per kill = C/E.

b. Sorties per kill = 1/E.

c. Aircraft losses per kill = A/E.

4. Weapon Selection. NAVMOR chooses "optimal" weapons using two subroutines:

a. Subroutine OPTIMA selects (by total enumeration) the best combination of weapon load N and number of passes n (recall we combine these into subscript $r = (N,n)$). This is done separately for each of the three measures of effectiveness, for example,

$$\min_{r} \frac{C}{E}(i,j,k,d,\ell,m,r) = \frac{C}{E}(i,j,k,\ell,m,r^*)$$

where the best $r^*(i,j,k,d,\ell,m)$ includes n^* and N^* (as a function of all subscripts except r).

b. Then subroutine SELECT optimizes over weapon type j and delivery tactic d (again by complete enumeration and again separately for each of the three measures of effectiveness, C/E, 1/E and A/E.

$$\min_{j,d} \frac{C}{E}(i,j,k,\ell,m,d,r^*) = \frac{C}{E}(i,k,\ell,m)$$

yielding $j^*(i,k,\ell,m)$, the best weapon and $d^*(i,k,\ell,m)$, the best delivery tactic.

The resulting best weapon is compared manually for each of the three MOE's. If all three agree, then

that weapon is automatically selected, otherwise a weapon is chosen which is satisfactory for each MOE although possibly not optimal for any of them. This process is not clearly defined. The result is, for <u>each</u> combination of subscripts

i = aircraft type

k = target type

ℓ = ceiling level

and m = mission type

we have a selected

j^* = weapon

d^* = delivery tactic

and r^* = weapon load and number of passes.

Note that all of these selections are made without reference to any constraints on sortie availability, required damage, relative target values, or total cost. Each selection looks at only one kind of sortie.

5. <u>Sortie Allocation</u>. The major decision input to NAVMOR is the allocation of sorties. This allocation is made by subjective judgment of a group of Naval officers based on the scenario which gives overall aircraft availability. The result is

$S(i,k,\ell,m)$ = number of sorties of type i,k,ℓ,m to be flown

where the sortie of type i,k,ℓ,m is required to use the previously selected j^*, d^*, r^*.

6. Final Bookkeeping. Finally the selected sortie allocations can be used to compute

a. Expected total number of weapons of each type used.

b. Expected number of targets of each type killed.

c. Expected total cost of weapons, sorties, and attrition.

as measures of the weapons requirement, overall effectiveness, and overall cost.

7. Miscellaneous Notes.

a. The major difference between NAVMOR and the other models considered in this report is the lack of optimization of the sortie allocation. NAVMOR instead attempts to select a sortie allocation which represents a typical allocation which might result from selection by Naval officers in a combat situation. This has the advantage of not being overly optimistic about actual decision making and of retaining an element of making sense which optimization models sometimes lose if their constraints are not carefully formulated. There is also, however, the disadvantage that only one allocation is considered and finally evaluated--hence we do not know how good this allocation is as compared to others or how much better the results could be if the allocation were changed to some other equally reasonable values.

b. The weapons used, and the cost and effectiveness of the allocation are outputs from the NAVMOR model. If any of these are unsatisfactory, it is not generally obvious how the sortie allocation should be changed to improve the situation. It is not possible to put restrictions on these outputs in advance and guarantee that the final solution will satisfy the restrictions.

E. NAVMOR PLUS (1976). The NAVMOR PLUS model [7,8] is an attempt to improve the NAVMOR procedure by adding a sortie optimization facility to it. The model is designed to use the same data inputs as the NAVMOR model, so the results are directly comparable, and NAVMOR output can be used as a starting point in the NAVMOR PLUS optimizations. The NAVMOR PLUS model is a linear programming model which allocates sorties to minimize cost subject to constraints on sortie and weapon availability and on effectiveness achieved by the allocation.

1. Decision Variables. The decision variables for the NAVMOR PLUS are

$Z_{ijk\ell}$ = expected number of (post attrition) sorties actually flown by

aircraft of type $i = 1,\ldots,IT$ against
targets of type $j = 1,\ldots,JT$ using
weapons of type $k = 1,\ldots,KT$ under
weather ceiling $\ell = 1,\ldots,LT$.

2. Objective Function. Two different objective functions can be used:

a. Minimize weighted cost per kill,

$$\min \mathrm{CPK}(Z) = \sum_{ijk\ell} \frac{C_{ijk\ell}}{P_{ijk\ell}} Z_{ijk\ell}$$

where $C_{ijk\ell}$ = cost of sortie of type $ijk\ell$

and $P_{ijk\ell}$ = expected number of type j targets killed by one sortie of type $ijk\ell$.

b. Minimize total cost,

$$\min \mathrm{COST}(Z) = \sum_{ijk\ell} C_{ijk\ell} Z_{ijk\ell}$$

For each of these two objectives the sortie cost $C_{ijk\ell}$ may include any of weapon costs, overhead and maintenance costs, and attrition costs.

3. Weapon Usage Constraints. Upper and lower bounds on usage of each weapon type may be specified:

$$C_1 : WL_k \leq \sum_{ij\ell} N_{ijk\ell} Z_{ijk\ell} \leq WU_k \qquad \forall\ k$$

where WL_k and WU_k are the bounds and $N_{ijk\ell}$ = number of weapons used in a sortie of type $ijk\ell$. These constraints guarantee that weapons in the current inventory will be used and also may limit weapon usage where production capability is limited.

4. Expected Effectiveness Constraints. The weapons planner can guarantee the effectiveness level of the weapons allocation by imposing bounds on expected targets killed.

$$C_2 : EL_j \leq \sum_{ik\ell} P_{ijk\ell}\, Z_{ijk\ell} \leq EU_j \qquad \forall\, j\,.$$

5. Constraints on Preattrition Sorties Allocated to Targets.

$$C_3 : \sum_{k} G_{ijk\ell} \cdot U_{ijk\ell} \cdot Z_{ijk\ell} = (\leq)\ ST_{ij\ell} \qquad \forall\, i,\, j,\, \ell$$

where $U_{ijk\ell}$ = number of sorties which a single aircraft could fly if no attrition

$C_{ijk\ell}$ = an attrition factor

$ST_{ij\ell}$ = bound on total number of sorties of type i, j, ℓ which may be flown for all weapons k.

The purpose of these constraints is not clearly explained, but seems to be related to a desire for the model to exactly reproduce the NAVMOR allocation under some circumstances (using = in the constraint and ST given by the NAVMOR allocation).

6. General Constraints on Preattrition Sorties.

$$C_4 : \sum_{jk} G_{ijk\ell}\, U_{ijk\ell}\, Z_{ijk\ell} = (\leq)\ AT_{i\ell} \qquad \forall\, i, \ell$$

These constraints are like the previous ones, except the

model is allowed to optimize the target type j and the weapon type k. These constraints place availability limits on the total numbers of sorties by aircraft type and weather ceiling conditions.

7. Typical NAVMOR-PLUS Investigations. Two kinds of problems are typically solved in the NAVMOR-PLUS model: The first is

$$LP_1: \quad \min CPK(Z)$$

$$ST\{C_1, C_2, C_3\}$$

which can be used to reproduce the allocation of the basic NAVMOR procedure as a starting point for further investigations.

The second model is

$$LP_2: \quad \min COST(Z)$$

$$ST\{C_1, C_2, C_4\}$$

which minimizes total cost and does not attempt to reproduce the NAVMOR allocation.

8. Solution Method. The NAVMOR-PLUS model yields straightforward (if somewhat large) linear programs which are solved using the Univac "Functional Mathematical Programming System."

9. Miscellaneous Notes.

a. NAVMOR delivery tactics are preselected in advance as the best possible tactic (min cost per kill) which can be used in the given weather ceiling condition.

b. Similarly, weapon load (but not weapon type) per sortie is preselected to minimize cost per kill. Both a. and b. are consistent with NAVMOR procedures.

c. The aggregation of cost per kill values for different targets in the CPK(Z) objective function implicitly assumes that the benefit from a kill is independent of target type. This is clearly unrealistic unless target units are carefully scaled. This objective function seems to be used primarily to reproduce the NAVMOR solution rather than as a final goal.

d. The linear treatment of target damage in the effectiveness constraints is subject to the same limitations as in the RAND linear programming model.

e. The source report for this model description was not a final report--NAVMOR PLUS should be viewed as a model which is still under development, although a current version is implemented in NAVCOSSACT [8].

III. COMPONENTS FOR MUNITIONS PROGRAMMING MODELS

In this section we review the models described in Section II with the goal of comparing the way they treat several of the basic components of munitions programming models. We will simultaneously suggest some areas for possible extensions.

A. Costs.

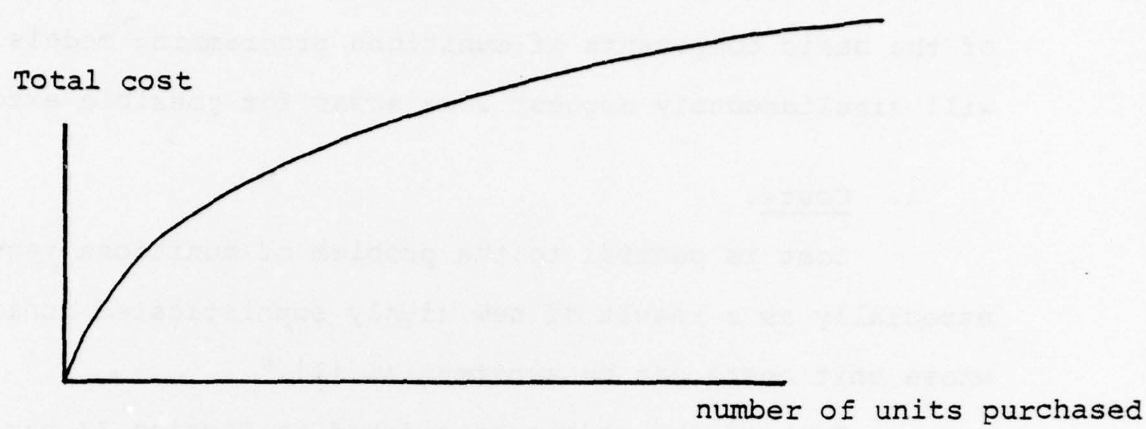
Cost is central to the problem of munitions requirements, especially as a result of new highly sophisticated munitions whose unit costs can be substantial [1].

Each of the models considered in Section II has a cost segment. In the RAND LP and the NAVMOR PLUS models cost is the objective function. The TAC RESOURCER model has budget constraints which involve cost computations. The RAND NLP model and the NAVMOR model only consider cost as an input in the pre-optimization selection of weapons for each aircraft-target combination. Both treatments of cost--as either an objective or a constraint--seem reasonable. In addition models can be imagined where cost would show up in both, e.g. minimize total cost subject to a constraint on attrition cost (and other constraints, of course).

There are several costs which may be included:

1. Munitions Cost. The cost of the munitions expended is included in every munitions model. This cost is generally considered to be a linear function of the number of weapons used. For new munitions the linear function may not adequately measure

the front end design, and acquisition costs which typically lead to concave cost functions having decreasing average cost as development expenses are spread over a larger number of units.



The implication of such a cost function for munitions programming is that new systems may be not cost effective if purchased in small quantities even though larger buys would be cost effective. The implication for optimization modeling is that minimizing a concave cost function yields a difficult optimization problem--one which may have local minimum solutions which are not global and for which large problems cannot be routinely solved. This is perhaps the reason why such cost structures are not found in existing munitions requirement models.

2. Sortie Costs. In addition to munitions costs, there are costs associated with each sortie flown. These include operating costs (e.g. fuel) and perhaps also pro-rata shares

of maintenance and/or basing costs. The decision of what to include and what to exclude does not seem simple, and few guidelines are given in the reports reviewed. The situation becomes even more complex when we consider that certain costs (e.g. maintenance, training) may depend on the total number of different systems supported as well as on the number of sorties planned for each system. Costs of this sort have not been included in any of the models surveyed.

3. Attrition Costs. For sorties where the delivery aircraft is vulnerable to enemy defensive forces it is important to consider the cost of attrition. As indicated in Section II the various models do so to greater or lesser degrees. Attrition cost generally includes aircraft replacement cost and crew replacement cost (pilot training etc.). In many models it is computed as a constant times the number of sorties. Several of the references include cautions about double-counting so considerable care is called for in including attrition cost in such a model.

B. Scenarios--Contingencies.

The RAND LP model is unique among those considered in that it considers several contingencies simultaneously. Each contingency has its own target list, and the weapons requirement is computed to be adequate for any one of the contingencies (but not necessarily more than one).

All other models considered in this report work on a single contingency. Multiple threats have to be analyzed by making separate runs and then somehow combining the answers from these runs. Needless to say, it is not at all obvious how this combining ought to be done.

C. Limits on Sorties Flown.

Squadron capabilities in the given scenario place limits on the numbers of sorties which can be flown by each aircraft type. Some models also subdivide these according to delivery condition (day/night, weather) to ensure a realistic situation. These limitations are readily incorporated as constraints in the resulting optimization models.

Several models also put limitations on the number of sorties by aircraft type and target type. These would not seem to be a reflection of the scenario, but rather to arise in an ad hoc manner forcing the optimization solutions to look more reasonable. (If you do not like the current solution, then add a constraint to make it infeasible.)

The availability of sorties may or may not be affected by aircraft attrition. Models which do not decrease available sorties by attrition factors assume that sufficient reserve aircraft and crews are available to keep the squadron at full force. If this is not the case, then the models should reduce sortie availability by realistic attrition factors.

D. Time Phasing.

The TAC RESOURCER model is the only one considered here which explicitly incorporates time phasing in which the scenario is broken down into several time periods. The primary advantage of time phasing over the all-at-one-time approach of the other models is that the model can be more realistic:

1. A target list that changes over time can be included.
2. In particular, targets of some types can be reconstituted in a later time period after having been defeated earlier.
3. Sortie availability can be more accurately modeled--especially in cases where this availability changes with time (reinforcement).

The single disadvantage of time phasing is that it makes the optimization model much larger, and hence much harder (perhaps impossible) to solve.

The TAC RESOURCER model allocates suboptimally--one time period at a time--so the target list at time t does not affect our allocation at time $t-1$. The sortie availabilities do vary with time, and the remaining target list at time t does depend on earlier sortie allocations. If the time periods are not too short, this probably is a fairly accurate reflection of what actually happens in combat where next week's target list is not known until next week and thus does not influence today's allocations.

In the NAVMOR procedure time is also considered, although in a slightly less satisfactory way, by separating the problem

into two disjoint time segments each having its own target list and sortie availability. Targets in the second time period do not, however, seem to depend on our allocations in the first period.

E. Munitions Selection.

Given a sortie using aircraft i against target j with weather ℓ (etc.) we have to decide which munition (k) will be delivered by the sortie. Three of the models surveyed in this report (RANDLP, TAC RESOURCER, and NAVMOR PLUS) include all possible munitions in the model and then the best munition is chosen by the optimization procedure. In the remaining two models (RAND NLP and NAVMOR) the "optimal" munition is preselected using a least cost per expected kill criterion for each aircraft-target combination. Then only these most cost effective weapons are used in the sortie allocation.

This preselection of munitions is clearly suboptimal, since sortie limitations cannot influence munition selection and under some circumstances, this may be a serious problem. For example, the least cost per expected kill munition may be a cheap but unsophisticated munition which requires more sorties than are available in the specified scenario. The result is a total effectiveness which may not meet requirements. By selecting a more expensive and more effective munition (with higher cost per kill), required target damage can be obtained within the limit on available sorties.

The advantage of preselection is that the resulting model has fewer decision variables and hence is easier to solve.

F. Attrition.

The models surveyed differ in their treatment of sortie attrition. If the model is being used to select munitions based (in part) on attrition of aircraft, it seems obvious that the attrition factors should depend on the munitions. In particular these factors should be computed in a way that accurately reflects the differences between long and short range munitions.

In the RAND NLP model, attrition does not seem to be explicitly included at all. It may appear in the costs used in the munition pre-selection process, but this is not discussed in the surveyed report.

The other models all include attrition factors dependent on all subscripts and thus (if the values are computed sensibly) meet the requirement stated above. As indicated earlier, the attrition may or may not influence availability of sorties.

G. Effectiveness Modeling.

Perhaps the greatest diversity in the models surveyed lies in the computation and use of target damage values. The models fall into three distinct groups. The first group (RAND NLP and TAC RESOURCER) uses target damage as the objective function--maximize the expected military worth of targets destroyed.

A crucial input which is required for the objective function of these two models is the relative military worth of a target in each of the target classes. We must decide the value of killing (say) a fuel dump as compared to that for a population center. These values are generally difficult to determine and even more difficult to defend. Both models also place constraints on the number of targets killed in each target class to ensure reasonable solutions (e.g. cannot kill more targets than exist).

The second class of models (RAND LP, NAVMOR PLUS) optimizies cost and uses constraints to assure that adequate damage is inflicted on each set of targets. This requires the model-user to specify required damage levels (perhaps both lower and upper bounds) for each class of target. These requirements are probably easier to set than the target military worth values required for the previous class of models. It is interesting that both of the models in the second class are linear, while both of the models in the first class are nonlinear.

NAVMOR stands in a class by itself, since it has neither constraints nor objective function.

A major issue in effectiveness modeling is whether non-linearities are required in the functions which compute expected target damage. The linear functions, such as in NAVMOR PLUS

$$\sum_{ik\ell} P_{ijk\ell} \, Z_{ijk\ell}$$

P = expected number of targets killed per sortie (constant)

Z = number of sorties (decision variable)

have the advantage of simplicity, ease of formulation and data specification, and ease of model solution.

The nonlinear functions, such as in TAC RESOURCER claim to represent reality more accurately (and must pay the resulting price in model complexity and difficulty of solution). The nonlinearities are introduced to more accurately model the following features of weapon-target engagements [3,4].

1. Target availability decreases as more sorties are launched. Some sorties may not detect live targets to attack.
2. For some target classes it may be difficult to distinguish live targets from dead ones. The result is that some already dead targets will be attacked again with no increase in effectiveness.
3. Remaining targets may be harder to kill either because they offer stiffer resistance or because mobile targets are dispersed.
4. Attacking forces may become less effective either because of attrition or because elements of surprise are no longer present.
5. The nonlinear functions arise from considering probabilistic (e.g. binomial) stochastic models of attack if some reasonable assumptions are used.

IV. CONCLUSIONS

A. Extensions.

Two areas in which extensions might make munitions programming models more accurate are cost and risk. Some possibilities for improving the cost functions in these models were presented in Section III.A.

All of the models presented in this report are probability models in that they compute the expected targets killed for the sortie allocation chosen. Any probability model involves an element of risk since the actual outcome will generally not equal the expected or average outcome. Under these circumstances it is appropriate to consider whether the models are sensitive to this risk. For example a constraint which requires expected targets killed (in some target class) to be greater than a required value, R, can be roughly interpreted as saying "half the time you will kill at least R but half the time you will kill less." The weapons planner might well desire a higher confidence than 50% that he will in fact achieve the effectiveness threshold of R.

Other uncertainties that are faced by the weapons planner and which a sophisticated model might include are the following.

1. The actual target list might differ from the postulated list implying that the selected munitions might not meet effectiveness requirements. This seems especially risky if the munitions chosen include a large proportion of special purpose munitions.

2. Various probability estimates may turn out to be incorrect --for example, attrition might be higher than expected.
3. Cost estimates may be uncertain (especially for new systems or those now in development).

Models for directly assessing these risks and developing munitions buys adequate to meet them will of necessity be more complex than current models. To ensure that the resulting models can be solved it will be essential to consider only the most important sources of uncertainty, and probably to aggregate some decisions which are now considered separately.

B. Summary.

Several models for determining air-to-ground munitions requirements have been reviewed in this report. They involve significant differences in degree of detail contained and in the nature of solution methods used. Some evaluations of these models along with suggestions for possible extensions have been included in the hope that such study may lead to improved models in the future.

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